Assignment 1.

This homework is due *Tuesday*, September 3.

There are total 30 points in this assignment. 27 points is considered 100%. If you go over 27 points, you will get over 100% for this homework and it will count towards your course grade (but not over 115%).

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper and *credit your collaborators*. Your solutions should contain full proofs. Bare answers will not earn you much.

This assignment covers section 1.1 in Bartle–Sherbert.

(1) (Exercise 1.1.5 in textbook) Prove the Distributive laws:
  (a) [3pt] $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$,
  (b) [3pt] $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(2) ($\sim$1.1.7) For each $n \in \mathbb{N}$, let $A_n = \{(n+1)k \mid k \in \mathbb{N}\}$.
  (a) [2pt] What is $A_2 \cap A_3$?
  (b) [3pt] Determine sets $\bigcup_{n=1}^{\infty} A_n$, $\bigcap_{n=1}^{\infty} A_n$.

(3) Let $f : A \to B$ and $E, F \subseteq A$.
  (a) [3pt] (Part of 1.1.14) Show that $f(E \cup F) = f(E) \cup f(F)$.
  (b) [3pt] (Part of 1.1.14) Show that $f(E \cap F) \subseteq f(E) \cap f(F)$.
  (c) [2pt] Show that not always $f(E \cap F) = f(E) \cap f(F)$. (*Hint:* to find a counter-example, you can start by picking E and F that do not intersect *at all.*)
  (d) [2pt] Show that not always $f(E \setminus F) \subseteq f(E) \setminus f(F)$. (*Hint:* to find a counter-example, you can start by picking $f(E)$ and $f(F)$ that *coincide.*)

(4) (Part of 1.1.15) [3pt] Let $f : A \to B$ and $G, H \subseteq B$. Prove that $f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H)$.
Comment. Compare to 3c.

(5) (1.1.22+) Let $f : A \to B$ and $g : B \to C$.
  (a) [3pt] Show that if $g \circ f$ is injective, then f is injective. Give an example that shows that g need not be injective.
  (b) [3pt] Show that if $g \circ f$ is surjective, then g is surjective. Give an example that shows that f need not be surjective.